



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 (a) Given that a is an integer, show that the system of equations

$$ax + 3y + z = 14,$$

$$2x + y + 3z = 0,$$

$$-x + 2y - 5z = 17,$$

has a unique solution and interpret this situation geometrically.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equations in part (a). [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x + 1.$$

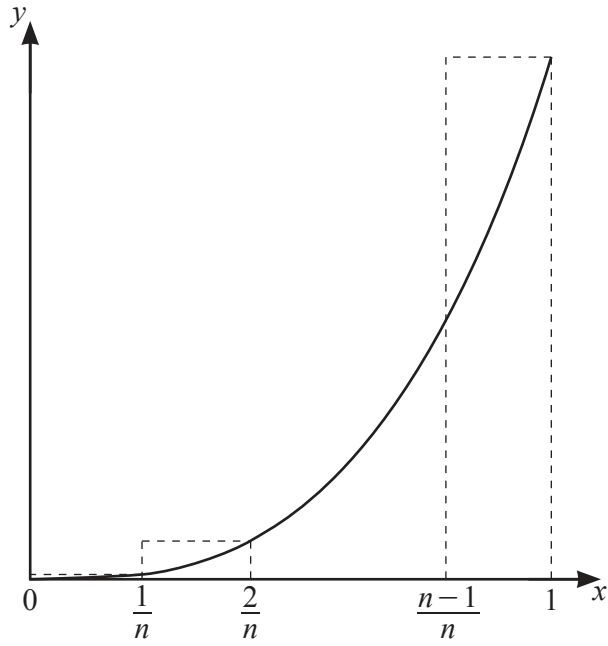
(a) Find the general solution for y in terms of x . [6]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

(b) State an approximate solution for large positive values of x . [1]

.....
.....
.....

3



The diagram shows the curve with equation $y = x^3$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 dx < U_n$, where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \tag{4}$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 Find the solution of the differential equation

$$\sin \theta \frac{dy}{d\theta} + y = \tan \frac{1}{2} \theta,$$

where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [9]

[You may use without proof the result that $\int \operatorname{cosec} \theta d\theta = \ln \tan \frac{1}{2} \theta$.]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A series of horizontal dotted lines for writing, spanning most of the page width.

5 (a) State the sum of the series $z + z^2 + z^3 + \dots + z^n$, for $z \neq 1$. [1]

.....

(b) Given that z is an n th root of unity and $z \neq 1$, deduce that $1 + z + z^2 + \dots + z^{n-1} = 0$. [2]

.....

(c) Given instead that $z = \frac{1}{3}(\cos \theta + i \sin \theta)$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3 \cos \theta - 1}{10 - 6 \cos \theta}. \quad [7]$$

.....

A series of horizontal dotted lines for writing, starting from below the page number and extending to the bottom of the page, just above the footer.

Dotted lines for writing.

(b) Use the characteristic equation of \mathbf{A} to find \mathbf{A}^3 . [4]

Dotted lines for writing.

7 (a) It is given that $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$.

Express $\cosh y$ in terms of x and hence show that $\sinh y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the first three terms in the Maclaurin's series for $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ in the form

$$\ln a + bx + cx^2,$$

where a, b and c are constants to be determined. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A series of horizontal dotted lines for writing.

8 The curve C has parametric equations

$$x = 2 \cosh t, \quad y = \frac{3}{2}t - \frac{1}{4} \sinh 2t, \quad \text{for } 0 \leq t \leq 1.$$

- (a) Find $\frac{dx}{dt}$ and show that $\frac{dy}{dt} = 1 - \sinh^2 t$. [3]

.....

.....

.....

.....

.....

.....

The area of the surface generated when C is rotated through 2π radians about the x -axis is denoted by A .

- (b) (i) Show that $A = \pi \int_0^1 \left(\frac{3}{2}t - \frac{1}{4} \sinh 2t \right) (1 + \cosh 2t) dt$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Dotted lines for writing answers.

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.